OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE
MATHEMATICS (MEI) 4763
Mechanics 3

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
None

Wednesday 27 January 2010
Afternoon

Duration: 1 hour 30 minutes

* 4 7 6 3 *

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 **(a)** **(i)** Write down the dimensions of density, kinetic energy and power. **[3]**

A sphere of radius r is moved at constant velocity v through a fluid.

(ii) In a viscous fluid, the power required is $6\pi\eta r v^2$, where η is the viscosity of the fluid.

Find the dimensions of viscosity. **[3]**

(iii) In a non-viscous fluid, the power required is $k\rho^{\alpha} r^{\beta} v^{\gamma}$, where ρ is the density of the fluid and k is a dimensionless constant.

Use dimensional analysis to find α, β and γ. **[6]**

(b) A rock of mass 5.5 kg is connected to a fixed point O by a light elastic rope with natural length 1.2 m. The rock is released from rest in a position 2 m vertically below O, and it next comes to instantaneous rest when it is 1.5 m vertically above O.

Find the stiffness of the rope. **[6]**

2 **(a)** A uniform solid hemisphere of volume $\frac{2}{3}\pi a^3$ is formed by rotating the region bounded by the x-axis, the y-axis and the curve $y = \sqrt{a^2 - x^2}$ for $0 \leqslant x \leqslant a$, through 2π radians about the x-axis.

Show that the x-coordinate of the centre of mass of the hemisphere is $\frac{3}{8}a$. **[5]**

(b) A uniform lamina is bounded by the x-axis, the line $x = 1$, and the curve $y = 2 - \sqrt{x}$ for $1 \leqslant x \leqslant 4$. Its corners are A (1, 1), B (1, 0) and C (4, 0).

(i) Find the coordinates of the centre of mass of the lamina. **[9]**

The lamina is suspended with AB vertical and BC horizontal by light vertical strings attached to A and C, as shown in Fig. 2. The weight of the lamina is W.

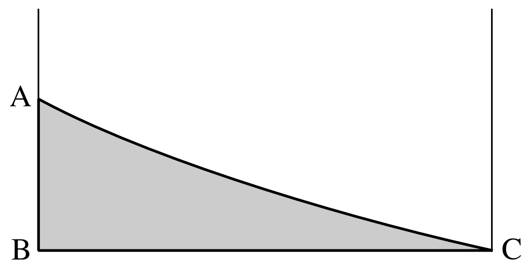

Fig. 2

(ii) Find the tensions in the two strings in terms of W. **[4]**

3 A particle P of mass 0.6 kg is connected to a fixed point O by a light inextensible string of length 1.25 m. When it is 1.25 m vertically below O, P is set in motion with horizontal velocity $6\,\text{m}\,\text{s}^{-1}$ and then moves in part of a vertical circle with centre O and radius 1.25 m. When OP makes an angle θ with the downward vertical, the speed of P is $v\,\text{m}\,\text{s}^{-1}$, as shown in Fig. 3.1.

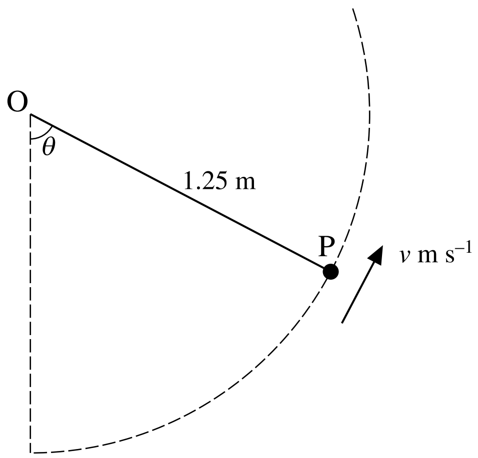

Fig. 3.1

(i) Show that $v^2 = 11.5 + 24.5\cos\theta$. **[3]**

(ii) Find the tension in the string in terms of θ. **[4]**

(iii) Find the speed of P at the instant when the string becomes slack. **[4]**

A second light inextensible string, of length 0.35 m, is attached to P, and the other end of this string is attached to a point C which is 1.2 m vertically below O. The particle P now moves in a horizontal circle with centre C and radius 0.35 m, as shown in Fig. 3.2. The speed of P is $1.4\,\text{m}\,\text{s}^{-1}$.

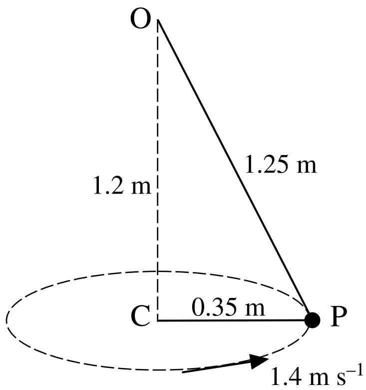

Fig. 3.2

(iv) Find the tension in the string OP and the tension in the string CP. **[7]**

[Question 4 is printed overleaf.]

4 Fig. 4 shows a smooth plane inclined at an angle of 30° to the horizontal. Two fixed points A and B on the plane are 4.55 m apart with B higher than A on a line of greatest slope. A particle P of mass 0.25 kg is in contact with the plane and is connected to A and to B by two light elastic strings. The string AP has natural length 1.5 m and modulus of elasticity 7.35 N; the string BP has natural length 2.5 m and modulus of elasticity 7.35 N. The particle P moves along part of the line AB, with both strings taut throughout the motion.

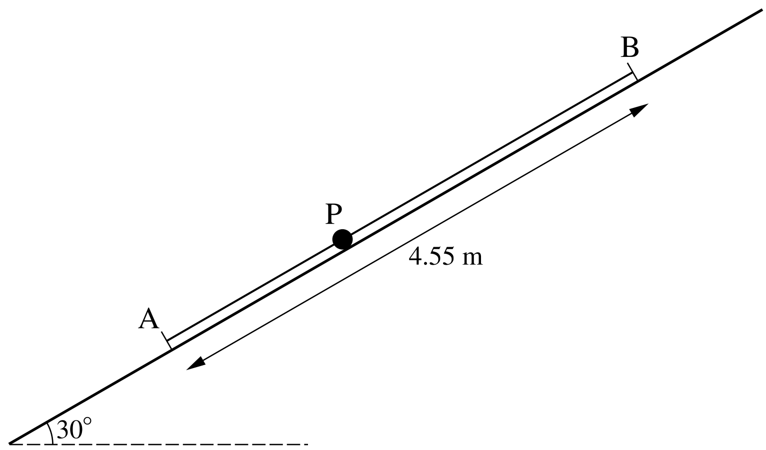

Fig. 4

(i) Show that, when AP = 1.55 m, the acceleration of P is zero. **[5]**

(ii) Taking AP = $(1.55 + x)$ m, write down the tension in the string AP, in terms of x, and show that the tension in the string BP is $(1.47 - 2.94x)$ N. **[3]**

(iii) Show that the motion of P is simple harmonic, and find its period. **[5]**

The particle P is released from rest with AP = 1.5 m.

(iv) Find the time after release when P is first moving *down* the plane with speed $0.2\ \text{m s}^{-1}$. **[5]**